

Lepton mass and mixing in a simple extension of the Standard Model based on T_7 flavor symmetry

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A simple Standard Model Extension based on T_7 flavor symmetry which accommodates lepton mass and mixing with non-zero θ_{13} and CP violation phase is proposed. At the tree- level, the realistic lepton mass and mixing pattern is derived through the spontaneous symmetry breaking by just one vacuum expectation value (v) which is the same as in the Standard Model. Neutrinos get small masses from one $SU(2)_L$ doublet and two $SU(2)_L$ singlets in which one being in $\underline{1}$ and the two others in $\underline{3}$ and $\underline{3}^*$ under T_7 , respectively. The model also gives a remarkable prediction of Dirac CP violation $\delta_{CP} = 172.598^\circ$ in both normal and inverted hierarchies which is still missing in the neutrino mixing matrix.

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I. INTRODUCTION

The discovery of neutrino mass is a great breakthrough for particle physics, and up to now, this is the unique evidence of New Physics. Neutrinos have tiny masses and this is

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probably related to the existence of a new mass scale in physics. Recently it has been shown that neutrinos can also play an important role in providing answer for the Baryon Asymmetry of Universe (BAU).

Theoretically, there exist various models describing the smallness of neutrino mass and large θ_{13} mixing¹. Among the possible extensions of the Standard Model (SM), probably the simplest one is the neutrino minimal SM which has been studied in Refs. [2–6]. However, these extensions do not provide a natural explanation for large mass splitting between neutrinos and the lepton mixing was not explicitly explained [7] .

There are five well-known patterns of lepton mixing [8], however, the Tri-bimaximal one proposed by Harrison-Perkins-Scott (HPS) [9–12]

$$U_{\text{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

seems to be the most popular and can be considered as a leading order approximation for the recent neutrino experimental data. Up to now, the absolute values of the entries of the lepton mixing matrix U_{PMNS} have not yet been determined exactly, however, their scales are given in Ref. [13]

$$|U_{\text{PMNS}}| = \begin{pmatrix} 0.801 \rightarrow 0.845 & 0.514 \rightarrow 0.580 & 0.137 \rightarrow 0.158 \\ 0.225 \rightarrow 0.517 & 0.441 \rightarrow 0.699 & 0.614 \rightarrow 0.793 \\ 0.246 \rightarrow 0.529 & 0.464 \rightarrow 0.713 & 0.590 \rightarrow 0.776 \end{pmatrix}. \quad (2)$$

The range of experimental values of neutrino mass squared differences and leptonic mixing angles are given in Ref. [14] as below

$$\begin{aligned} \sin^2 \theta_{12} &= 0.304 \pm 0.014, & \sin^2 \theta_{13} &= (2.19 \pm 0.12) \times 10^{-2}, \\ \Delta m_{21}^2 &= (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{23} &= 0.514_{-0.056}^{+0.055} \text{ (normal mass hierarchy),} \\ \sin^2 \theta_{23} &= 0.511 \pm 0.055 \text{ (inverted mass hierarchy),} \\ \Delta m_{32}^2 &= (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2, \text{ (normal mass hierarchy),} \\ \Delta m_{32}^2 &= (2.49 \pm 0.06) \times 10^{-3} \text{ eV}^2, \text{ (inverted mass hierarchy).} \end{aligned} \quad (3)$$

¹ The references for these models are mentioned in Ref. [1]

In fact, the models that successfully explain the experimental data are often mathematically complicate. An ideal physical model should be mathematically quite simple but successfully explains the experimental data and its physical parameters can be tested by the experiments in near future. This desired model, up to now, has not yet been effective because each model has its own advantages and disadvantages. To explain the specific neutrino mixings, it is simple to use discrete symmetry such as A_4, S_3, S_4 , etc. The use of non-abelian discrete symmetries to construct the models describing the lepton masses and mixings is a new method first proposed by E. Ma and G. Rajasekaran in 2001 [15]. In this treatment, there are various models which have been proposed, see for example A_4 [15–33], S_3 [34–74], S_4 [75–103], D_4 [104–114], T' [115–124], T_7 [125–129]. However, in all above mentioned papers, the fermion masses and mixings generated from non-renormalizable interactions or at loop levels but not at tree-level. The models involving only renormalizable interactions were implemented in our previous works [131–143] in which the discrete symmetries have been added to the 3-3-1 models. As we know the 3-3-1 model itself is an extension of the SM where the gauge group $SU(2)_L$ is extended to $SU(3)_L$. In order to overcome such limitations, we studied a neutrino mass model by adding the discrete symmetry S_4 to the SM which accommodates the realistic lepton mass, mixing with non-zero θ_{13} and CP violation phase at the tree- level with renormalizable interactions only [1].

In this paper, we construct a simple extension of the SM based on T_7 symmetry that leads to lepton mass, mixing with non-zero θ_{13} and CP violation phase². For this purpose, two $SU(2)_L$ doublets and two $SU(2)_L$ singlets are introduced. The result follows without perturbation and the number of scalars required to generate lepton masses are fewer than those in Ref. [1].

The future content of this paper reads as follows. In Sec. II we present the fundamental elements of the model and introduce necessary Higgs fields responsible for the lepton masses. We summarize the results in the section III. Finally, the appendices A and B provide in detail solutions for neutrino masses in the normal and the inverted hierarchies, respectively.

² We note that T_7 symmetry has not been previously considered in this kind of the model with the mentioned scenario. Furthermore, this model is different from our previous works [136, 138] because the 3-3-1 model (based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$) itself is an extension of the SM.

II. LEPTON MASS AND MIXING

The lepton content of the model, under $SU(2)_L \otimes U(1)_Y \otimes U(1)_X \otimes \underline{T}_7$ symmetries, is given in Tab. I. The charged lepton masses arise from the couplings of $\bar{\psi}_L l_{1R}$, $\bar{\psi}_L l_{2R}$ and

TABLE I: Lepton content of the model.

	ψ_L	$l_{(1,2,3)R}$	ν_R	ϕ	φ	χ	ζ
$SU(2)_L$	2	1	1	2	2	1	1
$U(1)_Y$	-1	-2	0	1	1	0	0
$U(1)_X$	1	1	0	0	-1	0	0
T_7	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{3}$	$\underline{3}^*$

$\bar{\psi}_L l_{3R}$ to scalars, where $\bar{\psi}_L l_{iL}$ ($i = 1, 2, 3$) transforms as 2 under $SU(2)_L$ and $\underline{3}^*$ under T_7 . In order to generate masses for charged leptons, we need only one $SU(2)_L$ Higgs doublets (ϕ) lying in $\underline{3}$ under T_7 , as given in Tab.I.

The Yukawa interactions read

$$\begin{aligned}
-\mathcal{L}_l &= h_1(\bar{\psi}_L \phi)_{\underline{1}} l_{1R} + h_2(\bar{\psi}_L \phi)_{\underline{1}'} l_{2R} + h_3(\bar{\psi}_L \phi)_{\underline{1}''} l_{3R} + H.c \\
&= h_1(\bar{\psi}_{1L} \phi_1 + \bar{\psi}_{2L} \phi_2 + \bar{\psi}_{3L} \phi_3) l_{1R} \\
&+ h_2(\bar{\psi}_{1L} \phi_1 + \omega^2 \bar{\psi}_{2L} \phi_2 + \omega \bar{\psi}_{3L} \phi_3) l_{2R} \\
&+ h_3(\bar{\psi}_{1L} \phi_1 + \omega \bar{\psi}_{2L} \phi_2 + \omega^2 \bar{\psi}_{3L} \phi_3) l_{3R} + H.c.
\end{aligned} \tag{4}$$

In this work we impose only the breaking $T_7 \rightarrow Z_3$ in charged lepton sector, and this happens with the first alignment, i.e, $\langle \phi \rangle = (\langle \phi_1 \rangle, \langle \phi_1 \rangle, \langle \phi_1 \rangle)$ under T_7 , where

$$\langle \phi_1 \rangle = (0 \quad v)^T. \tag{5}$$

With the vacuum expectation value (VEV) of ϕ_1 in Eq. (5), the mass Lagrangian for the charged leptons can be written in matrix form as

$$-\mathcal{L}_l^{\text{mass}} = (\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L}) M_l (l_{1R}, l_{2R}, l_{3R})^T + H.c, \tag{6}$$

where

$$M_l = \begin{pmatrix} h_1 v & h_2 v & h_3 v \\ h_1 v & \omega^2 h_2 v & \omega h_3 v \\ h_1 v & \omega h_2 v & \omega^2 h_3 v \end{pmatrix}. \tag{7}$$

The mass matrix M_l in Eq. (7) is diagonalized :

$$U_L^\dagger M_l U_R = \begin{pmatrix} \sqrt{3}h_1 v & 0 & 0 \\ 0 & \sqrt{3}h_2 v & 0 \\ 0 & 0 & \sqrt{3}h_3 v \end{pmatrix} \equiv \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (8)$$

where

$$m_e = \sqrt{3}h_1 v, \quad m_\mu = \sqrt{3}h_2 v, \quad m_\tau = \sqrt{3}h_3 v, \quad (9)$$

and

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad U_R = 1. \quad (10)$$

The Yukawa couplings $h_{1,2,3}$ in charged lepton sector are defined:

$$h_1 = \frac{m_e}{\sqrt{3}v}, \quad h_2 = \frac{m_\mu}{\sqrt{3}v}, \quad h_3 = \frac{m_\tau}{\sqrt{3}v}. \quad (11)$$

The experimental values for masses of the charged leptons are given in [14]:

$$m_e \simeq 0.510998928 \text{ MeV}, \quad m_\mu = 105.6583715 \text{ MeV}, \quad m_\tau = 1776.86 \text{ MeV} \quad (12)$$

It follows that $h_1 \ll h_2 \ll h_3$. Furthermore, if we choose³ the VEV $v \sim 100 \text{ GeV}$ then

$$h_1 \sim 10^{-6}, \quad h_2 \sim 10^{-4}, \quad h_3 \sim 10^{-2}, \quad (13)$$

i.e, in the model under consideration, the hierarchy between the masses for charged-leptons can be achieved if there exists a hierarchy between Yukawa couplings h_i ($i = 1, 2, 3$) in charged-lepton sector as given in Eq. (13). We note that the masses of charged leptons are self-separated by only one Higgs triplet ϕ (the same as in the SM), and this is a good feature of the T_7 group. We remind that the models with the other discrete symmetry groups need more than one Higgs scalar in the charged lepton sector.

The neutrino masses arise from the couplings of $\bar{\psi}_L \nu_R$ and $\bar{\nu}_R^c \nu_R$ to scalars, where $\bar{\psi}_L \nu_R$ transforms as 2 under $\text{SU}(2)_L$ and $\underline{1} \oplus \underline{1}' \oplus \underline{1}'' \oplus \underline{3} \oplus \underline{3}^*$ under T_7 ; $\bar{\nu}_R^c \nu_R$ transform as 1 under

³ In the SM, the Higgs VEV v is 246 GeV, fixed by the W boson mass and the gauge coupling $m_W^2 = \frac{g^2}{4} v_{weak}^2$. In the model under consideration $M_W^2 \simeq \frac{3}{2} g^2 v^2$. Therefore, we can identify $v_{weak}^2 = 6v^2 = (246 \text{ GeV})^2$. It follows $v \simeq 100 \text{ GeV}$.

$SU(2)_L$ and $\underline{3} \oplus \underline{3}^* \oplus \underline{3}^*$ under T_7 . Note that $\underline{3} \otimes \underline{3} \otimes \underline{3}$ has two invariants and $\underline{3} \otimes \underline{3} \otimes \underline{3}^*$ has one invariant under T_7 . In order to generate mass for neutrinos, we additionally introduce one $SU(2)_L$ doublet (φ) and two $SU(2)_L$ singlets (χ, ζ), respectively, put in $\underline{1}$, $\underline{3}$ and $\underline{3}^*$ under T_7 as given in Tab. I. We note that the $U(1)_X$ symmetry forbids the Yukawa terms of the form $(\bar{\psi}_L \tilde{\phi})_{\underline{3}_s} \nu_R$ and yield the expected results in neutrino sector, and this is interesting feature of X -symmetry. It is also interesting to note that φ contributes to the Dirac mass matrix, χ and ζ contribute to the Majorana mass matrix of the right-handed neutrinos. In fact, there exist no one-dimensional representation in $\underline{3} \otimes \underline{3}$ under T_7 . Hence, ζ put in $\underline{3}^*$ will be responsible for a realistic neutrino spectrum without any perturbation and soft breaking in both lepton and neutrino sectors. This feature is different from the one in Ref. [130]. It needs to note that φ contributes to the Dirac mass matrix in the neutrino sector, χ and ζ contribute to the Majorana mass matrix of the right-handed neutrinos. The interesting feature of X -symmetry is to prevents the unwanted interaction of the form $(\bar{\psi}_L \tilde{\phi})_{\underline{3}_s} \nu_R$ and provides the expected results in the neutrino sector.

In this work we impose that the breaking $T_7 \rightarrow \{\text{identity}\}$ must be taken place, i.e, T_7 is completely broken in neutrino sector. This can be achieved within each case below.

- (1) A new $SU(2)_L$ singlet χ lies in $\underline{3}$ under T_7 with the VEV is given by $\langle \chi \rangle = (0, \langle \chi_2 \rangle, 0)^T$ under T_7 , where

$$\langle \chi_2 \rangle = v_\chi. \quad (14)$$

- (2) Another singlet ζ lies in $\underline{3}^*$ under T_7 with the VEV is given by $\langle \zeta \rangle = (\langle \zeta_1 \rangle, \langle \zeta_2 \rangle, \langle \zeta_3 \rangle)^T$ under T_7 , i.e. $\langle \zeta_1 \rangle \neq \langle \zeta_2 \rangle \neq \langle \zeta_3 \rangle \neq 0$, where

$$\langle \zeta_i \rangle = u_i \quad (i = 1, 2, 3). \quad (15)$$

The neutrino Yukawa interactions are given by

$$\begin{aligned} -\mathcal{L}_\nu &= x(\bar{\psi}_L \tilde{\varphi})_{\underline{3}^*} \nu_R + \frac{y}{2}(\bar{\nu}_R^c \chi)_{\underline{3}^*} \nu_R + \frac{z}{2}(\bar{\nu}_R^c \zeta)_{\underline{3}^*} \nu_R + H.c \\ &= x(\bar{\psi}_{1L} \tilde{\varphi} \nu_{1R} + \bar{\psi}_{2L} \tilde{\varphi} \nu_{2R} + \bar{\psi}_{3L} \tilde{\varphi} \nu_{3R}) \\ &\quad + \frac{y}{2}[(\bar{\nu}_{2R}^c \chi_3 + \bar{\nu}_{3R}^c \chi_2) \nu_{1R} + (\bar{\nu}_{3R}^c \chi_1 + \bar{\nu}_{1R}^c \chi_3) \nu_{2R} + (\bar{\nu}_{1R}^c \chi_2 + \bar{\nu}_{2R}^c \chi_1) \nu_{3R}] \\ &\quad + \frac{z}{2}(\bar{\nu}_{1R}^c \zeta_2 \nu_{1R} + \bar{\nu}_{2R}^c \zeta_3 \nu_{2R} + \bar{\nu}_{3R}^c \zeta_1 \nu_{3R}) + H.c. \end{aligned} \quad (16)$$

The neutrino mass Lagrangian are given as

$$\begin{aligned}
-\mathcal{L}_\nu^{mass} = & xv(\bar{\nu}_{1L}\nu_{1R} + \bar{\nu}_{2L}\nu_{2R} + \bar{\nu}_{3L}\nu_{3R}) \\
& + \frac{y}{2}(v_\chi\bar{\nu}_{3R}^c\nu_{1R} + v_\chi\bar{\nu}_{1R}^c\nu_{3R} + v_\chi\bar{\nu}_{2R}^c\nu_{1R} + v_\chi\bar{\nu}_{1R}^c\nu_{2R}) \\
& + \frac{z}{2}(u_2\bar{\nu}_{1R}^c\nu_{1R} + u_3\bar{\nu}_{2R}^c\nu_{2R} + u_1\bar{\nu}_{3R}^c\nu_{3R}) + H.c.
\end{aligned} \tag{17}$$

We can rewrite in the matrix form

$$\begin{aligned}
-\mathcal{L}_\nu^{mass} = & \frac{1}{2}\bar{\chi}_L^c M_\nu \chi_L + H.c., \quad \chi_L \equiv \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}, \quad M_\nu \equiv \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \\
\nu_L^c = & (\nu_{1L}^c \quad \nu_{2L}^c \quad \nu_{3L}^c)^T, \quad \nu_R = (\nu_{1R} \quad \nu_{2R} \quad \nu_{3R})^T,
\end{aligned} \tag{18}$$

where the Dirac neutrino mass matrix (M_D) and the right-handed Majorana neutrino mass matrix (M_R) are given by

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, \quad M_R = \begin{pmatrix} N_2 & 0 & b \\ 0 & N_3 & 0 \\ b & 0 & N_1 \end{pmatrix}, \tag{19}$$

with

$$a = v_\varphi x, \quad b = v_\chi y, \quad N_i = u_i z \quad (i = 1, 2, 3). \tag{20}$$

The seesaw mechanism generates small masses for neutrinos is given by

$$M_{\text{eff}} = -M_D M_R^{-1} M_D^T = \begin{pmatrix} A_1 & 0 & B \\ 0 & A_3 & 0 \\ B & 0 & A_2 \end{pmatrix}, \tag{21}$$

where

$$A_1 = \frac{a^2 N_1}{b^2 - N_1 N_2}, \quad A_2 = \frac{a^2 N_2}{b^2 - N_1 N_2}, \quad A_3 = -\frac{a^2}{N_3}, \quad B = \frac{a^2 b}{N_1 N_2 - b^2}. \tag{22}$$

The matrix M_{eff} in Eq. (21) has three exact eigenvalues given by

$$\begin{aligned}
m_1 &= \frac{1}{2} \left(A_1 + A_2 - \sqrt{(A_1 - A_2)^2 + 4B^2} \right), \quad m_2 = A_3, \\
m_3 &= \frac{1}{2} \left(A_1 + A_2 + \sqrt{(A_1 - A_2)^2 + 4B^2} \right),
\end{aligned} \tag{23}$$

and the corresponding eigenstates are

$$U_\nu = \begin{pmatrix} \frac{K}{\sqrt{K^2+1}} & 0 & \frac{1}{\sqrt{K^2+1}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{K^2+1}} & 0 & -\frac{K}{\sqrt{K^2+1}} \end{pmatrix}, \quad (24)$$

where

$$K = \frac{A_1 - A_2 - \sqrt{(A_1 - A_2)^2 + 4B^2}}{2B}, \quad (25)$$

and $A_{1,2}, B$ are given in Eq. (22).

The lepton mixing matrix is then expressed as

$$U = U_L^\dagger U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1+K}{\sqrt{K^2+1}} & 1 & \frac{1-K}{\sqrt{K^2+1}} \\ \frac{K+\omega^2}{\sqrt{K^2+1}} & \omega & \frac{1-K\omega^2}{\sqrt{K^2+1}} \\ \frac{K+\omega}{\sqrt{K^2+1}} & \omega^2 & \frac{1-K\omega}{\sqrt{K^2+1}} \end{pmatrix}, \quad (26)$$

where K is defined in Eq.(25). Comparing the lepton mixing matrix in Eq. (26) and the standard parametrization ⁴ in Ref. [14] yields:

$$s_{13}e^{-i\delta} = \frac{1}{\sqrt{3}} \frac{1-K}{\sqrt{K^2+1}}, \quad (27)$$

$$t_{12}^2 = \left| \frac{\sqrt{K^2+1}}{1+K} \right|^2, \quad (28)$$

$$t_{23}^2 = \left| \frac{1-K\omega^2}{1-K\omega} \right|^2. \quad (29)$$

In the case K being real numbers, Eqs. (27) and (29) imply $\theta_{23} = 45^\circ$ and $\delta = 0$. As we know, the recent experimental data imply $\delta \neq 0$. To overcome this, we will consider K as a complex variable. Substituting $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ into Eqs. (27), (28) and (29) we obtain:

$$s_{13} = \frac{1}{\sqrt{3}} \frac{[(k_1 - 1)^2 + k_2^2]^{1/2}}{\alpha^{1/4}}, \quad (30)$$

$$t_{12}^2 = \frac{\alpha^{1/2}}{(1+k_1)^2 + k_2^2}, \quad (31)$$

$$t_{23}^2 = 1 - \frac{2\sqrt{3}k_2}{1+k_1+k_1^2+k_2^2+\sqrt{3}k_2}, \quad (32)$$

⁴ In fact, the Majorana phases do not contribute to neutrino oscillations so they will be ignored for the rest of this work.

where

$$\alpha = (1 + k_1^2 - k_2^2)^2 + 4k_1^2 k_2^2, \quad (33)$$

and k_1 and k_2 being the real and imaginary parts of K , respectively.

On the other hand, from Eq.(27), we get:

$$e^{-i\delta} = \frac{1}{s_{13}\sqrt{3}} \frac{1-K}{\sqrt{K^2+1}} \equiv \cos \delta - i \sin \delta, \quad (34)$$

with

$$\begin{aligned} \cos \delta &= (1 + 2k_1 - k_1^2 - k_2^2 - \sqrt{\alpha}) \beta, \\ \sin \delta &= \{k_2^2 - 1 + k_1(1 - k_1 + k_1^2 + k_2^2) + (1 - k_1)\sqrt{\alpha}\} \beta, \end{aligned} \quad (35)$$

where

$$\beta = \frac{\alpha^{1/4} \sqrt{-1 - k_1^2 + k_2^2 + \sqrt{\alpha}}}{\sqrt{2} \sqrt{(k_1 - 1)^2 + k_2^2} [(-1 - k_1^2 + k_2^2)\sqrt{\alpha} + k_1^4 + (k_2^2 - 1)^2 + 2k_2^2(1 + k_2^2)]}, \quad (36)$$

which is satisfying the relation $\sin^2 \delta + \cos^2 \delta = 1$ with all k_1, k_2 .

The neutrino mass spectrum can be the normal hierarchy ($|m_1| \simeq |m_2| < |m_3|$), the inverted hierarchy ($|m_3| < |m_1| \simeq |m_2|$) or nearly degenerate ($|m_1| \simeq |m_2| \simeq |m_3|$). The mass ordering of neutrino depends on the sign of Δm_{23}^2 which is currently unknown. However, some tight upper limits on the total neutrino mass $\sum m_\nu$ have given by the recent studies. For example, the total mass of three degenerate neutrinos was given by Planck satellite mission [144], $\sum m_\nu < 0.72$ eV (95% CL) by using Planck TT+lowP data, and $\sum m_\nu < 0.49$ eV (95% CL) by using Planck TT,TE,EE+lowP data. While the improved constraints are given by adding the baryon acoustic oscillation (BAO) measurements [145], i.e., $\sum m_\nu < 0.21$ eV (95% CL) and $\sum m_\nu < 0.17$ eV (95% CL), respectively. Another upper limit was given in Ref. [146], $\sum m_\nu < 0.113$ eV (95% CL).

As will see, in the model under consideration, the two possible signs of Δm_{23}^2 correspond to two types of neutrino mass spectrum as well as the values of the atmospheric neutrino mixing angle θ_{23} can be provided.

Combining Eq. (30) with the experimental values of θ_{13} given in Ref. [14] as shown in Eq.(3), we have a solution as follow⁵:

$$k_2 = -\frac{1}{2} \sqrt{(8.03468 - 4k_1)k_1 - 4.03468 + 2\sqrt{0.069663 + (0.139025k_1 - 0.139326)k_1}}. \quad (37)$$

⁵ There exist four mathematical solutions, however, these solutions differ only by the sign of $m_{1,2,3}$ which has no effect on the neutrino oscillation experiments.

Next, from Eqs. (37) and (32) with the experimental values of θ_{23} in Eq.(3), we get two solutions⁶:

$$k_1 = 0.690532, \quad k_2 = -0.0350532, \quad K = 0.690532 - 0.0350532i, \quad (38)$$

and the lepton mixing matrix in (26) then takes the form

$$|U| \simeq \begin{pmatrix} 0.803441 & 0.57735 & 0.147986 \\ 0.437621 & 0.57735 & 0.709451 \\ 0.405089 & 0.57735 & 0.689859 \end{pmatrix}, \quad (39)$$

which is consistent with constraint in Eq.(2). Now, substituting $k_{1,2}$ from (38) in to (31) yields⁷ $t_{12}^2 = 0.516381$ (or $t_{12} = 0.718597$), i.e, $\theta_{12} \simeq 35.7^\circ$. It follows $\cos \delta = -0.991667$, $\sin \delta = 0.128827$, i.e, $\delta \simeq 172.598^\circ$.

Combining (25) and the values of K in (38), we obtain

$$A_1 = A_2 - (0.753905 + 0.108377i)B. \quad (40)$$

A. Normal spectrum ($\Delta m_{23}^2 > 0$)

Substituting A_1 from (40) into (23) and combining with the two experimental constraints on squared mass differences of neutrinos for the normal spectrum as shown in (3), i.e, $\Delta m_{21}^2 = 7.53 \times 10^{-5} \text{ eV}^2$, $\Delta m_{32}^2 = 2.44 \times 10^{-3} \text{ eV}^2$, we get the analytical expressions of $A_2, B, m_{1,2,3}$ (in [eV]) given in Appendix A.

By using the upper limits on neutrino mass [144–146] we can restrict $A_3 \leq 0.72 \text{ eV}$. However, in the normal spectrum case in (A), $A_3 \in [0.0087, 0.03] \text{ eV}$ or $A_3 \in [-0.03, -0.0087] \text{ eV}$ are good regions of A_3 that can reach the realistic neutrino mass hierarchies. With $m_2 \in [0.0087, 0.03] \text{ eV}$, $m_{1,2,3}$ as functions of $A_3 = m_2$ are plotted in Fig.1. This figure shows that there exist allowed regions of the parameter A_3 where either normal or quasi-degenerate neutrino masses spectrum is achieved. The quasi-degenerate mass hierarchy⁸ is obtained when $A_3 \in [0.03 \text{ eV}, +\infty)$ or $A_3 \in (-\infty, -0.03 \text{ eV}]$ ($|A_3|$ increases but must be

⁶ Here we only consider one case because another value has no effect on the neutrino oscillation experiments.

⁷ $\theta_{12} \simeq 35.7^\circ$ obtained from the model is an acceptable prediction.

⁸ There is no clear limits between neutrino mass hierarchies by the recent experimental results on neutrino oscillations

small enough because of the scale of $m_{1,2,3}$). The normal mass hierarchy will be obtained if $A_3 \in [0.0087, 0.03] \text{ eV}$ or $A_3 \in [-0.03, -0.0087] \text{ eV}$. The total neutrino masses in the model under consideration $\sum_{i=1}^3 m_i$ and $\sum_{i=1}^3 |m_i|$ with $m_2 \in [0.0087, 0.05] \text{ eV}$ is depicted in Fig.2.

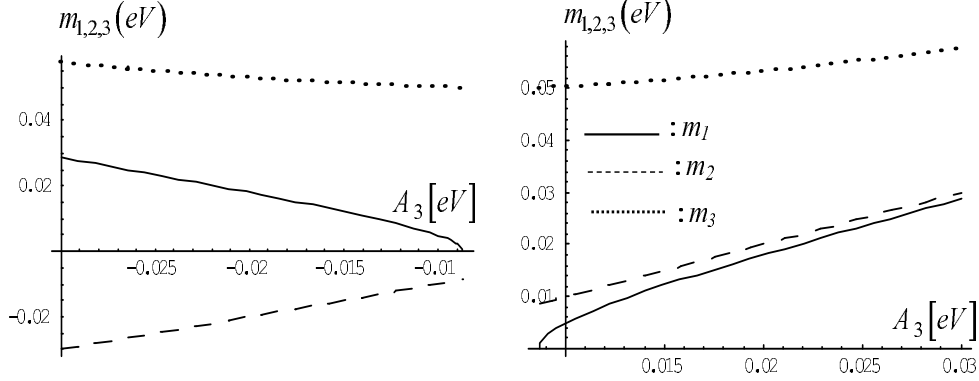


FIG. 1: $m_{1,2,3}$ as functions of A_3 in the normal spectrum with $A_3 \in (-0.03, -0.0087) \text{ eV}$ (left) and $A_3 \in (0.0087, 0.03) \text{ eV}$ (right).

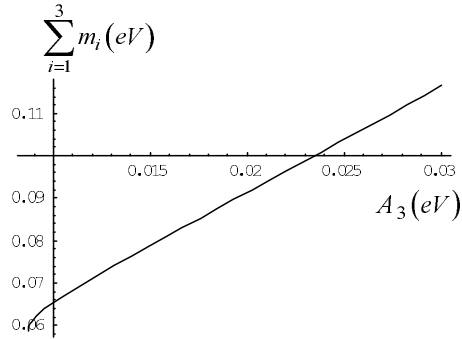


FIG. 2: The sum $\sum_{i=1}^3 m_i$ as a function of A_3 with $A_3 \in (0.0087, 0.03) \text{ eV}$ in the normal spectrum.

It is easily to obtain the effective mass $\langle m_{ee} \rangle$ governing neutrinoless double beta decay [147–152] $\langle m_{ee} \rangle = |\sum_{i=1}^3 U_{ei}^2 m_i|$, $m_\beta = \{\sum_{i=1}^3 |U_{ei}|^2 m_i^2\}^{1/2}$ by combining the expressions (26), (38), (A1), (A2) and (A3), the values of m_{ee} , m_β and m_{light} are plotted in Fig.3 together with m_1 with $A_3 \in (0.0087, 0.03) \text{ eV}$. We also note that in the normal spectrum, $m_1 \approx m_2 < m_3$ so $m_{light} = m_1$ given in (A2) is the lightest neutrino mass.

To get explicit values of the model parameters, we assume $A_3 \equiv m_2 = 10^{-2} \text{ eV}$, which is safely small. Then the other neutrino masses and the other parameters are explicitly given in Tab. II.

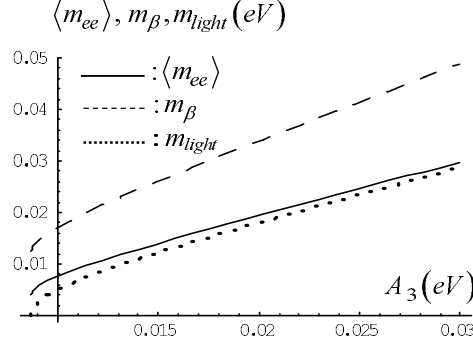


FIG. 3: m_{ee} , m_β and m_{light} as functions of A_3 with $A_3 \in (0.00867, 0.05)$ eV in the normal spectrum.

TABLE II: The model parameters in the case $A_3 = 10^{-2}$ eV in the normal spectrum

Parameters [eV]	The derived values
A_1	$0.0357232 + 0.00100894i$
A_2	$0.0196452 - 0.00100894i$
B	$-0.0212715 + 0.000381301i$
m_1	0.00496991
m_3	0.0503984
$\sum m_i^I$	0.0653683
$\langle m_{ee}^I \rangle$	0.00761271
m_β^I	0.0171627

Now, comparing Eqs. (22) and derived values in Tab. II we get the relations:

$$\begin{aligned}
 N_1 &= -(142.621 + 4.02808i)a^2, & N_2 &= (-78.4315 + 4.02808i)a^2, \\
 N_3 &= -100a^2, & b &= (-84.9243 + 1.52231i)a^2.
 \end{aligned} \tag{41}$$

or

$$|N_1|/|b| = 1.67979, \quad |N_2|/|b| = 0.924614, \quad |N_3|/|b| = 1.17733, \tag{42}$$

$$|N_1/a^2| = 142.678, \quad |N_2/a^2| = 78.5348, \quad |N_3/a^2| = 100, \quad |b/a^2| = 84.938, \tag{43}$$

i.e., N_1 , N_2 , N_3 and b have the same order of magnitude, and approximately two orders of magnitude of a^2 .

B. Inverted spectrum ($\Delta m_{23}^2 < 0$)

Substituting A_1 in (40) into (23) and combining with the experimental constraints on squared mass differences of neutrinos for the inverted spectrum as shown in (3), i.e, $\Delta m_{21}^2 = 7.53 \times 10^{-5} \text{ eV}^2$, $\Delta m_{32}^2 = -2.49 \times 10^{-3} \text{ eV}^2$, we get a solution (in [eV]) given in Appendix B.

In the inverted spectrum, with the solution in (B), $A_3 \in (0.055, 0.085) \text{ eV}$ or $A_3 \in [-0.085, -0.055] \text{ eV}$ are good regions of A_3 that can reach the inverted neutrino mass hierarchies. The absolute values $|m_{1,2,3}|$ as functions of $A_3 = m_2$ are plotted in Fig. 4 in which $A_3 \in [0.055, 0.085] \text{ eV}$. This figure shows that the quasi-degenerate mass hierarchy is obtained when $A_3 \in [0.085 \text{ eV}, +\infty)$ or $A_3 \in (-\infty, -0.085 \text{ eV}]$. The inverted mass hierarchy will be obtained if $|A_3| \in [0.055, 0.085] \text{ eV}$. The total neutrino masses $\sum_{i=1}^3 m_i^I$ and $\sum_{i=1}^3 |m_i^I|$ with $A_3 \in [0.055, 0.085] \text{ eV}$ is depicted in Fig.5.

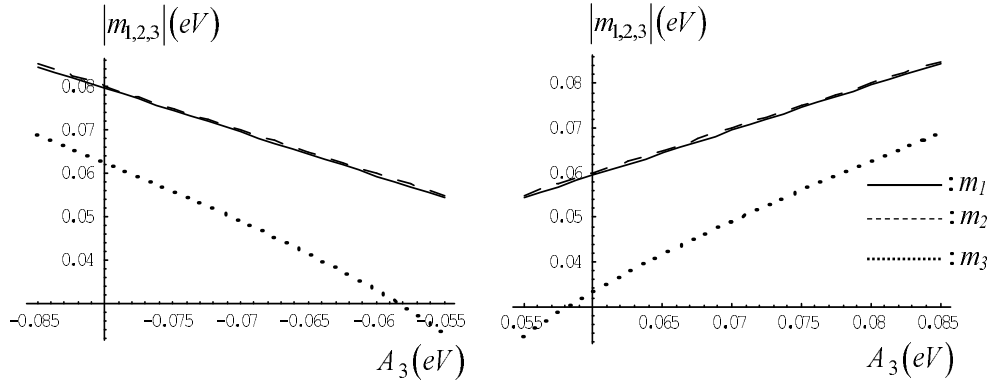


FIG. 4: $|m_{1,2,3}|$ as functions of A_3 in the inverted spectrum with $A_3 \in (-0.085, -0.055) \text{ eV}$ (left) and $A_3 \in (0.055, 0.085) \text{ eV}$ (right).

In the inverted spectrum, the effective mass $\langle m_{ee}^I \rangle$ governing neutrinoless double beta decay $\langle m_{ee}^I \rangle$ and m_β^I together with m_3 are plotted in Fig.6 with $A_3 \in [0.055, 0.085] \text{ eV}$ by combining the expressions (26), (38), (B1), (B2) and (B3). In this case $m_{light}^I = m_3$ given in Eq. (A3) is the lightest neutrino mass.

To get explicit values of the model parameters, we assume $A_3 \equiv m_2 = 6 \times 10^{-2} \text{ eV}$. The other neutrino masses and the other parameters are explicitly given in Tab. III.

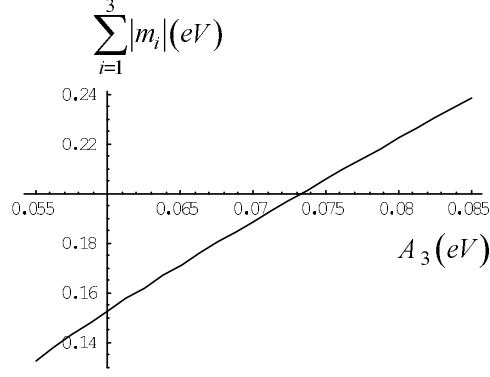


FIG. 5: The sum $\sum_{i=1}^3 m_i^I$ as a function of A_3 with $A_3 \in (0.055, 0.085)$ eV in the inverted spectrum.

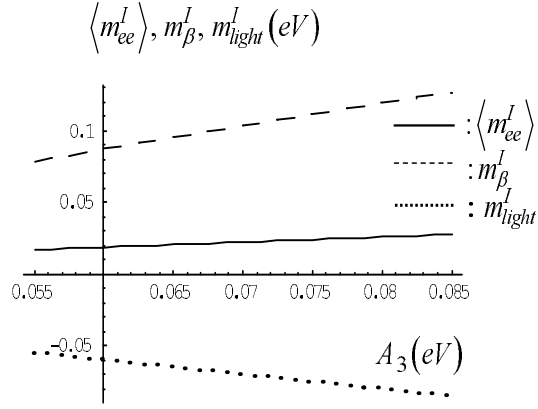


FIG. 6: m_{ee} , m_{β} and m_{light} as functions of A_3 with $A_3 \in (0.055, 0.085)$ eV in the inverted spectrum.

Comparing Eqs. (22) and derived values in Tab. III yields:

$$\begin{aligned} N_1 &= (21.0986 - 0.292525i)a^2, & N_2 &= (25.7602 + 0.292525i)a^2, \\ N_3 &= -16.6667a^2, & b &= (-6.16732 + 0.110552i)a^2. \end{aligned} \quad (44)$$

or

$$|N_1|/|b| = 3.42082, \quad |N_2|/|b| = 4.17648, \quad |N_3|/|b| = 2.70198, \quad (45)$$

$$|N_1/a^2| = 21.1006, \quad |N_2/a^2| = 25.7618, \quad |N_3/a^2| = 16.6667, \quad |b/a^2| = 6.16831, \quad (46)$$

i.e., N_1 , N_2 , N_3 and b have the same order of magnitude, and approximately one orders of magnitude of a^2 .

TABLE III: The model parameters in the case $A_3 = 6 \times 10^{-2}$ eV in the inverted spectrum

Parameters [eV]	The derived values
A_1	$-0.0417327 + 0.000578609i$
A_2	$-0.0509532 - 0.000578609i$
B	$-0.0121988 + 0.00021867i$
m_1	-0.0593692
m_3	-0.0333167
$\sum m_i^I$	0.0326858
$\langle m_{ee}^I \rangle$	0.0190284
m_β	0.08723

III. CONCLUSIONS

We have proposed a simple Standard Model extension based on T_7 flavor symmetry which accommodates lepton mass, mixing with non-zero θ_{13} and CP violation phase. The spontaneous symmetry breaking in the model is imposed to obtain the realistic lepton mass and mixing pattern at the tree- level with renormalizable interactions. In difference from other discrete groups, the T_7 flavor group requires only one VEV ($\langle \phi_1 \rangle = v$) which, the same as in the SM, is enough for production of the charged lepton masses. The neutrinos get small masses from one $SU(2)_L$ doublet and two $SU(2)_L$ singlets in which one being in $\underline{1}$ and the two others in $\underline{3}$ and $\underline{3}^*$ under T_7 , respectively. The model also gives a remarkable prediction of Dirac CP violation $\delta_{CP} = 172.598^\circ$ in both normal and inverted spectrum which is still missing in the neutrino mixing matrix.

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Appendix A: Neutrino masses in the normal spectrum

$$\begin{aligned}
A_2 &= 8.44732 \times 10^{-9} \sqrt{\Gamma} + (8.13873 \times 10^{-6} + 2.91875 \times 10^{-7} i) \sqrt{\gamma \Gamma} \\
&\quad - (0.188476 + 0.0270942 i) \sqrt{\gamma' - 2\sqrt{\gamma}} + 7.14452 \times 10^{-6} A_3^2 \sqrt{\Gamma}, \\
B &= -0.5 \sqrt{\gamma' - 2\sqrt{\gamma}}, \\
m_1 &= -0.5 \sqrt{0.0023647 + 2A_3^2 - (2.27831 + 0.081706i) \sqrt{\gamma}} \\
&\quad + (0.188476 + 0.0270943i) \sqrt{\gamma' - 2\sqrt{\gamma}} + 8.44732 \times 10^{-9} \sqrt{\Gamma} \\
&\quad + (8.13873 \times 10^{-6} + 2.91875 \times 10^{-7} i) \sqrt{\gamma \Gamma} \\
&\quad - (0.188476 + 0.0270942i) \sqrt{\gamma' - 2\sqrt{\gamma}} + 7.14452 A_3^2 \sqrt{\Gamma}, \tag{A1}
\end{aligned}$$

$$m_2 = A_3, \tag{A2}$$

$$\begin{aligned}
m_3 &= 0.5 \sqrt{0.0023647 + 2A_3^2 - (2.27831 + 0.081706i) \sqrt{\gamma}} \\
&\quad + (0.188476 + 0.0270943i) \sqrt{\gamma' - 2\sqrt{\gamma}} + 8.44732 \times 10^{-9} \sqrt{\Gamma} \\
&\quad + (8.13873 \times 10^{-6} + 2.91875 \times 10^{-7} i) \sqrt{\gamma \Gamma} \\
&\quad - (0.188476 + 0.0270942i) \sqrt{\gamma' - 2\sqrt{\gamma}} + 7.14452 \times 10^{-6} A_3^2 \sqrt{\Gamma}. \tag{A3}
\end{aligned}$$

where

$$\begin{aligned}
\gamma &= (-1.4104 \times 10^{-7} + 1.01291 \times 10^{-8} i) + (0.00181524 - 0.000130366i) A_3^2 \\
&\quad + (0.76764 - 0.0551299i) A_3^4, \tag{A4}
\end{aligned}$$

$$\gamma' = (0.00207317 - 0.0000743489i) + (1.75343 - 0.0628823i) A_3^2, \tag{A5}$$

$$\begin{aligned}
\Gamma &= (7.32235 \times 10^{12} + 0.000183105i) + (6.19305 \times 10^{15} - 0.0625i) A_3 \\
&\quad - (7.05485 \times 10^{15} + 2.53004 \times 10^{14} i) \sqrt{\gamma}. \tag{A6}
\end{aligned}$$

Appendix B: Neutrino masses in the inverted spectrum

$$\begin{aligned}
A_2 &= 9.5457 \times 10^{-9} \sqrt{\Gamma_1} - (8.4778 \times 10^{-6} + 3.04035 \times 10^{-7}i) \sqrt{\gamma_1 \Gamma_1} \\
&\quad - (0.188476 + 0.0270942i) \sqrt{\gamma'_1 - 2\sqrt{\gamma_1}} - 7.44217 \times 10^{-6} A_3^2 \sqrt{\Gamma_1}, \\
B &= -0.5 \sqrt{\gamma'_1 - 2\sqrt{\gamma_1}}, \\
m_1 &= -0.5 \sqrt{(-0.0025653 + 2.71051 \times 10^{-20}i) + 2A_3^2 - (2.27831 + 0.081706i) \sqrt{\gamma_1}} \\
&\quad + (0.188476 + 0.0270942i) \sqrt{\gamma'_1 - 2\sqrt{\gamma_1}} - (8.4778 \times 10^{-6} + 3.04035 \times 10^{-7}i) \sqrt{\gamma_1 \Gamma_1} \\
&\quad + (9.5457 \times 10^{-9} - 7.44217 \times 10^{-6} A_3^2) \sqrt{\Gamma_1}, \tag{B1}
\end{aligned}$$

$$m_2 = A_3, \tag{B2}$$

$$\begin{aligned}
m_3 &= 0.5 \sqrt{(-0.0025653 + 2.71051 \times 10^{-20}i) + 2A_3^2 - (2.27831 + 0.081706i) \sqrt{\gamma_1}} \\
&\quad + (0.188476 + 0.0270942i) \sqrt{\gamma'_1 - 2\sqrt{\gamma_1}} - (8.4778 \times 10^{-6} + 3.04035 \times 10^{-7}i) \sqrt{\gamma_1 \Gamma_1} \\
&\quad + (9.5457 \times 10^{-9} - 7.44217 \times 10^{-6} A_3^2) \sqrt{\Gamma_1}, \tag{B3}
\end{aligned}$$

where

$$\begin{aligned}
\gamma_1 &= (1.4393 \times 10^{-7} - 1.03367 \times 10^{-8}i) - (0.00196923 - 0.000141425i) A_3^2 \\
&\quad + (0.76764 - 0.0551299i) A_3^4, \tag{B4}
\end{aligned}$$

$$\gamma'_1 = (-0.00224904 + 0.000080656i) + (1.75343 - 0.0628823i) A_3^2, \tag{B5}$$

$$\begin{aligned}
\Gamma_1 &= -7.94351 \times 10^{12} + (6.19305 \times 10^{15} - 0.0625i) A_3^2 \\
&\quad - (7.05485 \times 10^{15} + 2.53004 \times 10^{14}i) \sqrt{\gamma_1}. \tag{B6}
\end{aligned}$$

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